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William M. Sanders, Michael D. Richardson

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A SENSITIVITY ANALYSIS OF PARAMETER ESTIMATES IN BUCKINGHAM'S GRAIN SHEARING MODEL

William M. Sanders[a], Michael D. Richardson[a]

[a] Naval Research Laboratory, Stennis Space Center, MS 39529, USA

William M. Sanders, Naval Research Laboratory, Stennis Space Center, MS 39529, USA, fax (228) 688-5752, wsanders@nrlssc.navy.mil

Abstract: *A compilation of 54 sets of geoacoustic measurements of marine sediments was reviewed in order to bound estimates of parameters for use in Buckingham's grain shearing (GS) theory of acoustic propagation. These data, for unconsolidated sands (siliciclastic and carbonate) with grain sizes between 0.0156 mm and 0.57 mm, were all made in shallow water (maximum depth of 60 m) sites at diverse locales. In each data set, measurements of the speed and attenuation of the compressional wave at high frequency, the shear wave speed at 1 kHz, porosity, and bulk density are sufficient to calculate the three free parameters in GS theory (a material exponent, a compressional coefficient and a shear coefficient). The spread of the values calculated for GS parameters, combined with a sensitivity analysis, do not support the use of a single material exponent value for all sediments. Nor do they support the notion that the material exponent be 1, which would be the case if elastic and viscous forces at grain contacts are equal. Finally, it is suggested that, given the sensitivity to spot measurements, the GS parameters be estimated simultaneously from a full range of compressional and shear wave measurements.*

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1. INTRODUCTION

Buckingham's Grain Shearing (GS) theory was introduced[1] as an alternative to poro-elastic theories, such as Biot's[2], in an attempt to better match observed dispersion curves. The foundation of this alternative theory, that rigidity in the sediment is provided by grain-to-grain shearing, removes the necessity of describing an elastic frame, an artificial construct which must be described by unobservable elastic parameters. However, GS theory introduces three parameters, which are not in themselves observable, but are derived from other physical observations.

GS theory describes stick-slip events at grain contacts, which, rather than being truly impulsive, have, on average, some exponential decay, parameterized by a material exponent n. Compressional and shear waves propagate via these grain contacts with relative intensity parameterized by two elastic moduli, γ_p and γ_s respectively. These are not elastic moduli relating stress and strain rates, but are related to the rate and intensity of radial and translational shear events at grain contacts. Not directly measureable, they are derived from observed quantities.

Given a known porosity N, GS theory requires estimation of three unknown parameters (n, γ_p and γ_s). Buckingham has described a method of calculating these three values from measurements of compressional and shear wave speeds and attenuations[3]. First compressional sound speed and attenuation at a high frequency is used to calculate the material exponenent n. Then the shear speed at a low frequency is used to calculate both moduli, γ_p and γ_s. This method avoids use of the shear wave attenuation, a usually imprecise measurement seldom available.

2. GEOACOUSTIC DATA SET

The geoacoustic measurements were all made in situ using various versions of the In Situ Sediment geoAcoustic Measurement System (ISSAMS). The measurement system is described in Chapter 5 of Jackson and Richardson [4] and most of the geoacoustic and physical property data used in these analyses can also be found in that reference. The data base can be traced back to the original sources using references in [4]. All in situ geoacoustic measurements were made at 20-30 cm below the sediment-water interface over path lengths ranging from 30-100 cm. Compressional wave speed and attenuation were measure at either 38 or 58 kHz using time-of flight and amplitude of a 5- to 10-cycle pulsed sine waves propagating between identical radially-poled ceramic cylinders through sediment and a reference of seawater just about the sediment water interface. Shear wave speed was measured at 1 kHz using time-of-flight between bimorph ceramic benders mounted on the same diver-deployed or remotely-operated hydraulic systems. Shear wave attenuation was measured at selected sites using a 4-transducer transposition technique which calculates shear wave attenuation from waveform amplitudes measured using two transmit and two receiver transducers. This technique eliminates the need to measure transducer sensitivity or measure variable insertion losses. Multiple sediment cores were collected from each site to measure high frequency (400 kHz) compressional wave speed and attenuation and to provide data of sediment physical properties such as grain size distribution, sediment bulk density and porosity. All values of wave speed and attenuation as well as bulk sediment physical properties used in these analyses are averages from multiple deployments and multiple sediment cores collected at the same location (a roughly 25-m^2 area).

The variability among measured wave speeds and attenuation at a single location is generally thought to be equal or greater than the actual measurement error which is less than 1% for wave speeds and less than 10% for attenuation.

For this study we restrict ourselves to data from the 54 sandy sites where the microscopic stress relaxation mechanisms that are part of the Buckingham theory are most likely to be applicable. It seems unlikely that this particle-to-particle stress relaxation mechanism can be applied to muddy sediment where electrostatic repulsive and attractive forces and the adhesion of organic matter control particle-to-particle interactions and the flexure of clay particles may provide a dissipation mechanism.

3. SENSITIVITY ANALYSIS

Errors in estimates of Buckingham's parameters are driven by uncertainty in the measurements of the physical parameters from which they are calculated. Given the high correlation between porosity and grain size, only one need be included, and here it is porosity. Buckingham describes a process of calculating all parameters by first determining c_o as a function of porosity N from Wood's equation[5], calculating the spectral exponent n from c_p and α_p, and finally evaluating the GS moduli from the shear wave speed c_s. Hence, the error in estimating n involves errors in c_p, α_p and N. Estimates of the GS moduli additionally involve errors in measuring c_s. Assuming the errors in measurements are Gaussian and uncorrelated, the overall error in each estimate is the sum of the contributions from individual sources.

$$
\sigma_n^2 = \left(\frac{\partial n}{\partial c_p}\right)^2 \sigma_{c_p}^2 + \left(\frac{\partial n}{\partial \alpha_p}\right)^2 \sigma_{\alpha_p}^2 + \left(\frac{\partial n}{\partial N}\right)^2 \sigma_N^2
$$

$$
\sigma_{\gamma_p}^2 = \left(\frac{\partial \gamma_p}{\partial c_s}\right)^2 \sigma_{c_s}^2 + \left(\frac{\partial \gamma_p}{\partial c_p}\right)^2 \sigma_{c_p}^2 + \left(\frac{\partial \gamma_p}{\partial \alpha_p}\right)^2 \sigma_{\alpha_p}^2 + \left(\frac{\partial \gamma_p}{\partial N}\right)^2 \sigma_N^2 \tag{1}
$$

$$
\sigma_{\gamma_s}^2 = \left(\frac{\partial \gamma_s}{\partial c_s}\right)^2 \sigma_{c_s}^2 + \left(\frac{\partial \gamma_s}{\partial c_p}\right)^2 \sigma_{c_p}^2 + \left(\frac{\partial \gamma_s}{\partial \alpha_p}\right)^2 \sigma_{\alpha_p}^2 + \left(\frac{\partial \gamma_s}{\partial N}\right)^2 \sigma_N^2
$$

Here σ_θ^2 is the variance of the estimate of the parameter θ. The partial derivatives given in equation (1) are given in the annex. Although the variance of the errors in the measurement of physical and geoacoustic parameters vary for each data set, for this analysis, it was assumed that the coefficient of variation in each measurement was the same for all data sets. Hence the following variances were assumed.

$$\sigma_N = .02N$$
$$\sigma_{c_p}^2 = .01c_p$$
$$\sigma_{\alpha_p}^2 = .30\alpha_p \quad (2)$$
$$\sigma_{c_s}^2 = .10c_s$$

4. RESULTS

The values calculated for the material exponent n, compressional modulus γ_p and shear modulus γ_s are shown in Figs. 1 to 3, respectively. Fifteen data sets yielded negative values for n, an unrealistic, yet mathematically possible result, and are hereafter omitted from the analysis. For each value calculated from the remaining 39 data sets, error bars span plus or minus one standard deviation about the estimate. Each parameter is plotted as a function of porosity. Carbonate sands are plotted in cyan, siliciclastic in blue, and individual SAX99 data in red. Values derived from SAX99 used by Buckingham are green[6].
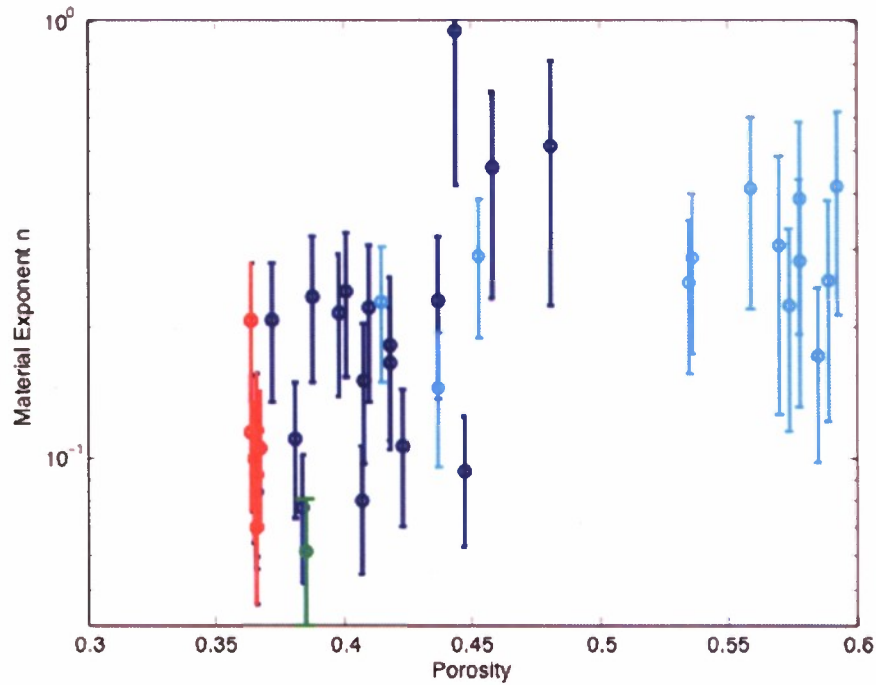

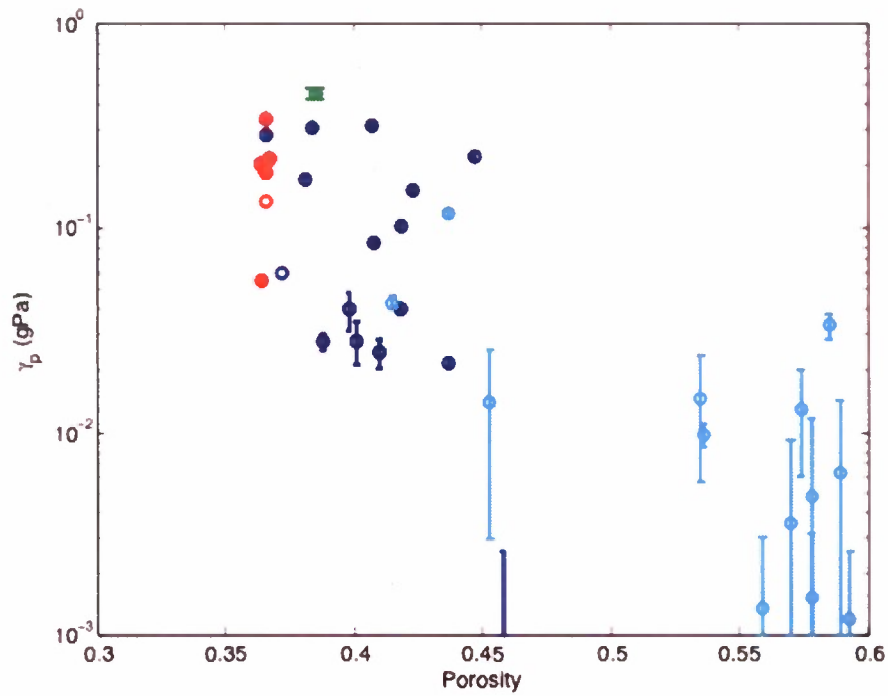Fig. 1 – Material exponent versus porosity.

Fig. 2 - Compressional modulus versus porosity.

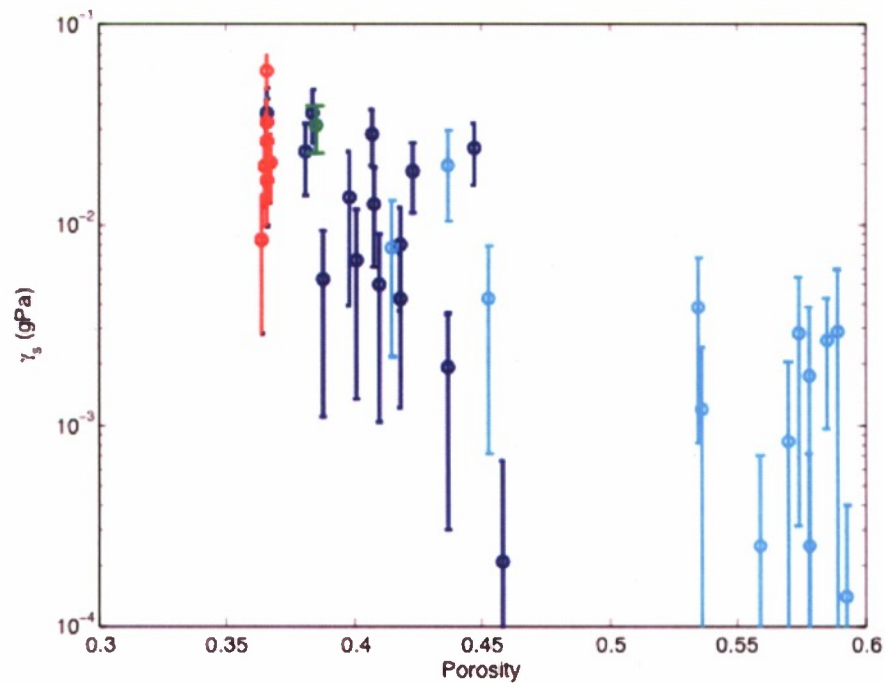**Fig. 3 - Shear modulus versus porosity.**

Although there is a wide spread in uncertainty in the material exponent shown in Fig. 1, there appears to be a dependence on porosity. The bulk of the values are of magnitude much less than one. This contradicts the notion that elastic and viscous forces at the grain contacts should be considered of equal importance, at least within context of the GS theory.

The apparent increase in value for n with increasing porosity contradicts Buckingham's assertion that one value for n can be used for all sediments[7]. Porosity is the determining factor in evaluating the low frequency limit of compressional sound speed c_0, given by the Wood equation. c_0, in turn, is used in calculating n. But there is obviously further dependence of n on porosity.

Assuming that estimates of porosity and compressional sound speed are accurate, and that inaccuracies in measuring compressional attenuation prevent accurate determination of n, an attempt was made to find a single value of n that resulted in values of α_p that is consistent with the entire data set. Specifically, if $\alpha_p^{\ h}$ is the attenuation required to give a hypothetical material exponent n^h and $\alpha_p^{\ i}$ is the measured attenuation from data set i, then a residual error can be computed.

$$\varepsilon^i = \alpha_p^i - \alpha_p^h$$

(3)

The value of n^h that minimized the mean square error for all data sets is $n^h = .14$. Adopting a lower value of n^h for all sediments ($n^h = .0851$), as Buckingham has suggested[3], implies that compressional attenuation used in the GS model is significantly lower than that observed in these data sets. However, this is consistent with his assertion that GS theory tends to giver lower bounds on attenuation values, in that it accounts only for intrinsic attenuation[3]. Other sources, such as scattering of sound by large-scale inhomogeneities are not accounted for in this theory.

Both grain-shear moduli, plotted in Figs. 2 and 3, also show strong dependence on porosity. Note the errors are relatively greater for carbonate sands than siliciclastic. Also the error in the estimate of γ_s is higher than that for γ_p. Generally, the largest source of error in the estimate of γ_p is due to errors in measurement of c_s, while that for the estimate of γ_s is due to errors in measurement of α_p.

The uncertainties given in the above figures are calculated for the method of evaluating the three GS parameters specified by Buckingham. However, it is postulated that alternative methods may yield more accurate estimates. For instance, rather than sequentially determining n, then the GS moduli, they may all be fit to observations simultaneously. The possible reduction in uncertainties in these estimates is discussed next.

5. BOUNDS ON ALTERNATIVE ESTIMATES OF BUCKINGHAM'S PARAMETERS

In a mapping **G** of a set of N_m model parameters to a set of observations **d**,

$$\mathbf{d} = \mathbf{G(m)}$$

(4)

the covariance of the A Posteriori errors, $\mathbf{C'_m}$, in the estimates of **m** can be calculated if all errors are assumed to be Gaussian and the mapping can be linearized about some estimate $\mathbf{m_0}$. In this case[8],

$$\mathbf{C}'_{\mathbf{m}} = \left(\mathbf{C}_{\mathbf{m}} + \mathbf{G}_1^{\mathrm{T}}(\mathbf{C}_{\mathbf{T}} + \mathbf{C}_{\mathbf{D}})^{-1}\mathbf{G}_1\right)^{-1}$$

(5)

Here $\mathbf{C}_{\mathbf{m}}$, is the covariance of the A Priori errors, $\mathbf{C}_{\mathbf{D}}$ is the covariance of the data (measurement errors), $\mathbf{C}_{\mathbf{T}}$ is the covariance of theoretical errors (due to mismatch between the forward model and reality), and $\mathbf{G}_1$ is given by the linearization

$$G_{ij} = \frac{\partial g(d_i)}{\partial m_j}$$

(6)

In this case, $\mathbf{m}$ is the set of GS parameters (n, γ_p, γ_s), d is the set of observations (N, c_p, α_p, c_s), the A Priori errors are given by the previous sensitivity analysis and the linearization provided by the partial derivatives previously derived. Measurement errors are assumed to be dominant, and theoretical errors are neglected.

In the case in which the three GS parameters are estimated simultaneously, the standard deviation of the error in estimating n (for the composite SAX99 data set) is only slightly reduced (by a factor of .95), while those for the two GS moduli are roughly halved. This is understandable, as Buckingham's sequential method uses the most reliable information to first calculate n, and the GS moduli are based on a more error prone measurement of the shear wave speed.

Another alternative was investigated, to see if uncertainties can be reduced more significantly. A second set of compressional wave measurements from the SAX99 experiment, provided by the diver deployed "attenuation array" gives c_p and α_p at 100 kHz[9]. Assuming similar variances in these estimates as with the 58 kHz data, the above error analysis again resulted in an insignificant decrease in the error in estimating n, about a halving of the error in estimating γ_s, but a reduction in the error in γ_p by a factor of .07.

6. CONCLUSIONS

For the GS theory to be validated, accurate and meaningful estimates of the three GS parameters must be achieved. It is not likely that this can be accomplished with field data. Laboratory conditions will likely provide better control over sediment properties and more accurate measurements. But this analysis of a large set of field observations, coupled with the sensitivity analysis, provides a first step in bounding the range of values that should be investigated.

The estimate of n is a critical first step and relies on accurate measurement of porosity, compressional sound speed and attenuation, all of which contribute significantly to the error. It is clear from this data set that a single value of n cannot be assumed for all sediments. But more accurate measurements are required in order to determine whether these GS parameters can be specified so that GS theory can be thoroughly validated.

7. ACKNOWLEDGEMENTS

REFERENCES

[1] **M. J. Buckingham**, Wave propagation, stress relaxation, and grain-to-grain shearing in saturated, unconsolidated marine sediments, *J. Acoust. Soc. Am.*, **108** (6), pp. 2796-2815, 2000.

[2] **M. A. Biot**, Theory of propagation of elastic waves in a fluid-saturated porous solid I. Low frequency range, *J. Acoust. Soc. Am.*, **28** (6), pp. 168-178, 1956.

[3] **M. J. Buckingham**, On pore-fluid viscosity and the wave properties of saturated granular materials including marine sediments, *J. Acoust. Soc. Am.*, **122** (3), pp. 1486-1501, 2007.

[4] **D. R. Jackson and M. D. Richardson**, *High-Frequency Seafloor Acoustics*, Springer, pp. 123-170, 2007.

[5] **A. B. Wood**, *A Textbook of Sound*, third ed., G. Bell, 1964.

[6] **M. J. Buckingham**, Compressional and shear wave properties of marine sediments: Comparisons between theory and data, *J. Acoust. Soc. Am.*, **117** (1), pp. 137-152, 2005.

[7] **B. T. Heffner and K. L. Williams**, Sound speed and attenuation measurements in unconsolidated glass-bead sediments saturated with viscous pore fluids, *J. Acoust. Soc. Am.*, **120** (5), pp. 2538-2549, 2006.

[8] **A. Tarantola**, *Inverse Problem Theory: Methods for Data Fitting and Parameter Estimation*, Elsevier, pp. 68-69, 1987.

[9] **E. I. Thorsos, K. L. Williams, N. P. Chotiros, J. Christoff, K. Commander, C. F. Greenlaw, D. V. Holliday, D. R. Jackson, J. L. Lopes, D. E. McGehee, J. E. Piper, M. D. Richardson, and D. Tang**, An overview of SAX99: Acoustic measurements, *IEEE J. Ocean. Eng.*, **26**, pp. 26-53, 2001.

ANNEXE

A1. PARTIAL DERIVATIVES

List of symbols

c_0	compressional sound speed in equivalent suspension
c_p	compressional wave speed in sediment
c_s	shear wave speed in sediment
K_0	Bulk modulus of equivalent suspension
K_g	Bulk modulus of grain
K_w	Bulk modulus of pore fluid
n	Strain-hardening index
N	Porosity
T	Arbitrary time constant
α_p	Compressional attenuation coefficient
χ	Dimensionless grain-shearing coefficient

γ_p Compressional wave grain-shearing modulus
γ_s Shear wave grain-shearing modulus
ρ_0 Bulk density of sediment
ω Angular Frequency

The partial derivatives of the Buckingham model parameters with respect to measured values are given below. For the sake of clarity, some derived parameters (X, χ) are given first. Then relevant expressions are given in terms of those derived parameters.

$$\frac{\partial n}{\partial \alpha_p} = \frac{1}{\pi c_p}\left\{\sin(n\pi) + 2X\sin^2\left(\frac{n\pi}{2}\right)\left[1 - \left(\frac{c_0}{c_p}\right)^2\left(1 - X^4\right)X^2\right]\right\}$$

$$\frac{\partial n}{\partial \alpha_p} = \frac{1}{\pi c_p}\left\{\sin(n\pi) + 2X\sin^2\left(\frac{n\pi}{2}\right)\left[1 + 2\left(\frac{c_0}{c_p}\right)^2\left(1 + X^2\right)\right]\right\}$$

$$\frac{\partial n}{\partial N} = \frac{1}{\pi X}\left(\frac{c_0}{c_p}\right)^2 \sin^2\left(\frac{n\pi}{2}\right)\left(1 + X^2\right)\left(\frac{K_0}{K_g} - \frac{K_0}{K_w}\right)$$

$$\frac{\partial X}{\partial n} = \frac{\pi X}{\sin(n\pi) + 2X\sin^2\left(\frac{n\pi}{2}\right)\left[1 + 2\left(\frac{c_0}{c_p}\right)^2\left(1 + X^2\right)\right]}$$

$$\frac{\partial \chi}{\partial n} = -\chi\left[\log(\omega T) + \frac{\pi}{2}\cot\left(\frac{n\pi}{2}\right)\right] + 2\left[\frac{\left(\frac{c_p}{c_0}\right)^2(\omega T)^{-n}\csc\left(\frac{n\pi}{2}\right) - 2X\chi\left(1 + X^2\right)}{\left(1 + X^2\right)^2}\right]\frac{\partial X}{\partial n}$$

$$\frac{\partial \gamma_s}{\partial n} = -\frac{\pi}{4}\rho_0 c_s^2 (\omega T)^{-n} \sin\left(\frac{n\pi}{2}\right) - \gamma_s \log(\omega T)$$

$$\frac{\partial \gamma_p}{\partial n} = \rho_0 c_0^2 \frac{\partial \chi}{\partial n} - \frac{4}{3}\frac{\partial \gamma_s}{\partial n}$$

$$\frac{\partial \gamma_s}{\partial \alpha_s} = 2\frac{\gamma_s}{c_s}$$

$$\frac{\partial \gamma_p}{\partial \alpha_s} = -\frac{4}{3}\frac{\partial \gamma_s}{\partial \alpha_s}$$